The Slope and the Curvature of the Yield Curve In Recession Forecasting

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Abstract

In this paper, we investigate the ability of two popular models to forecast the deviation of GDP from its long-run trend, i.e. inflationary and output gaps. In doing so, we exploit the information provided by the yield curve that is documented in the literature as a good predictor of economic activity. We combine and train our forecasting model using interest rates from Treasury Bills and Government Bond rates for the period 1976Q3 to 2011Q4, in conjunction with the quarterly real seasonally adjusted GDP for the same period. Our results show that we can achieve an overall forecasting accuracy of 80% on out-of-sample data. However, our main focus in this paper is to construct a forecasting model for the recessions. Perfect accuracy in recession forecasting is achieved in more than one of the created models. The forecasting performance of our model strengthens the conviction that the yield curve can be a useful and accurate predictive tool.

Keywords: support vector machines; yield curve, recession forecasting, GDP, machine learning.

1. Introduction

The yield curve is a graphical representation of the relationship between the maturity and the yield to maturity of bonds issued by a single debtor at a specific point in time. In the empirical literature, the behavior of the yield curve is associated with the Business Cycles. Business Cycles are measured by the fluctuations of real GDP from its long run trend (Stock and Watson, 1989). Although the term “cycle” implies a periodic phenomenon, the actual behavior of these fluctuations can prove unpredictable. However, there is significant evidence in the empirical literature that the yield curve can be an excellent indicator of future economic activity (e.g. an inverted yield curve is considered a sign of an upcoming recession or an output gap). In this study we will test the forecasting ability of two popular models: a) the Support Vector Machines model from Machine Learning and b) the well-established and widely used probit model from Econometrics. Both models are considered state-of-the-art in their domain. Even though, this paper builds on a wide range of previous research, to the best of our knowledge the SVM methodology has not previously been employed in forecasting the real GDP.

The interest and importance of forecasting future economic activity is not only academic. The ability to forecast within a reasonable margin of error upcoming output gaps is of great interest to policy makers and investors as well. Litterman (1986) applied a Bayesian Vector Autoregressive Model (BVAR) using data for the United States for the period from 1948Q1 to 1979Q3 to forecast the future economic activity. The variables included in the model were the real GDP, the GDP deflator, industrial investment, quarterly yields of government bonds, unemployment, and money supply. Litterman compared the results of his method to the forecasts of the Data Resources model, the Wharton model and the Chase Econometrics model and concluded that the BVAR outperformed the other models in most cases.

Estrella & Mishkin (1998) investigated the performance of a probit model to forecast U.S. recessions up to eight quarters in the future. They showed that when the interest rate spread was fed to a probit model it yielded more accurate results than other leading macroeconomic indicators. Christiansen (2012), tested the ability of the yield curve to forecast an economic event he terms “simultaneous recessions” (i.e. when recession occurs at least at the half countries in his dataset). He tested various probit models and concluded that the yield spreads of Germany and the U.S. are leading indicators of simultaneous recessions. Nyberg (2010) apart from testing the domestic term spread, he also applied the dynamic autoregressive probit model of Kauppi and Saikkonen (2008), for several lagged values of stock returns and the foreign term spread and proved that they can provide additional explanatory forecasting power for both Germany and the U.S. Wright (2006) focused mainly on the shape of the yield curve and proved that the probit models which forecast recessions and use both the level of the federal funds rate and the term spread give better out-of-sample predictive performance, than models with the term spread alone.

Gogas, Chionis and Pragidis (2009) confirmed the predictive ability of the yield curve for future economic activity in the European Union. They used the yield spreads, the EU15 unemployment rate and the stock indices of the London, Frankfurt, and Paris stock exchanges in several input setups fed in probit models. The authors concluded that the most accurate forecasts were produced by the spread between the 1 month and the 3 months maturity bonds for two quarters ahead prediction. The results showed that the addition of the unemployment rate to the input dataset did not affect the performance of the model, whereas the inclusion of financial indicators did improve its forecasting accuracy. Giacomini and Rossi (2005) provided some evidence that the predictive ability of the yield curve for growth has weakened since the 1980s and that is probably a stronger forecasting tool for recession cases only.

In this paper we mainly attempt to forecast U.S. recessions as introduced by Giacomini and Rossi (2005) and also, future economic activity employing an SVM framework using just the Treasury Bills and Government Bonds interest rates and the GDP (this is the dataset setup introduced by Piazzesi and Wei in 2004). The majority of Machine Learning techniques and methods need large data sets for model training. This is essential in schemes involving Neural Networks (NN) or Deep Learning (DL) architectures and this is the main reason we can find several attempts to use NN and DL methods within a financial context, though the same is not true for Macroeconomic forecasting where the sampling frequency prohibits the creation of long time series. However, long data sets are not a prerequisite to SVM based models. The training step can adequately treat small datasets yielding efficient forecasting models.

The remainder of this paper is organized as follows. Section 2 gives a brief description of the methodologies employed, in section 3 the dataset and the empirical results are presented and finally Section 4 concludes this contribution.

2. Methodology

2.1. Support Vector Machines

The Support Vector Machine is a machine learning method for classification and regression proposed by Vapnik in 1992. Machine Learning originates from the field of statistical learning. The basic intuition behind machine learning is to develop data driven models, fitted to the unique characteristics of the dataset under examination and contemporaneously enhance the ability to produce reliable forecasts of the evolution path of various phenomena. When it comes to SVM for classification, the basic idea is to find an optimal hyper plane that separates data points into two or more classes defined by a small subset of them, called Support Vectors. The support vector set is located in the dataset through a minimization procedure. Real life phenomena are often too complex to be modeled by linear models. The kernel-based solution for treating non-linear phenomena is to project the system into a higher dimensional space where the transformed dataset can be linearly modeled (in our case linearly classified). The kernel-based solution on SVM keeps the computational cost at minimum levels: the dataset is projected in an inner product space, where the projection is performed using only dot products within the original space through special “kernel” functions, instead of explicitly computing the mapping of each data point.

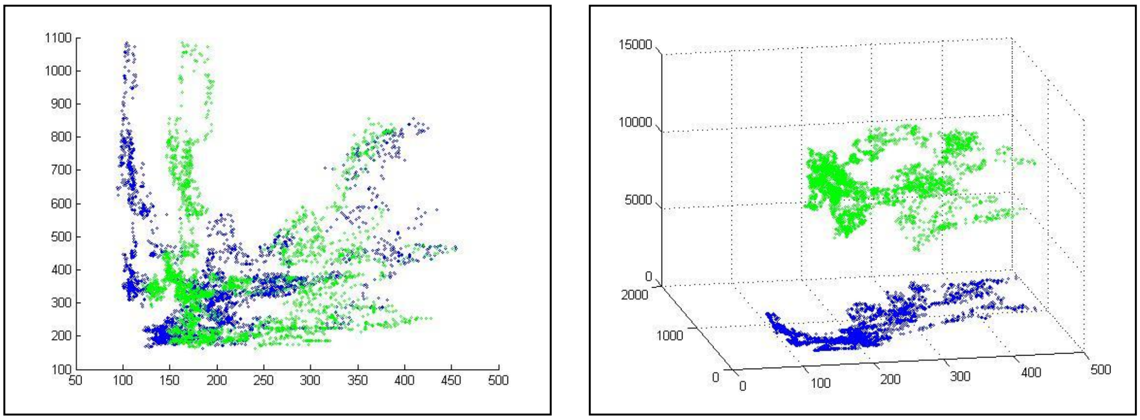

Figure 1. Data space (left), the data cannot be separated linearly. Feature space (right), the data points are projected into a higher space and the data can be separated linearly into two classes

One of the main advantages of SVM in comparison to other machine learning methods is that it can identify global minima and avoids selecting local ones, when reaching an optimal solution, (Vapnik, 1992). This feature is crucial to the generalization ability of the

SVM results as it produces accurate and reliable forecasts. The model is built in two steps: the training step and the testing step. In the training step, the largest part of the dataset is used for the estimation of the separating hyperplane (i.e. the detection of the Support Vectors that define the classification hyperplane); in the testing step, the generalization ability of the model is evaluated by checking the model's performance in the small subset that was left aside in the first step.

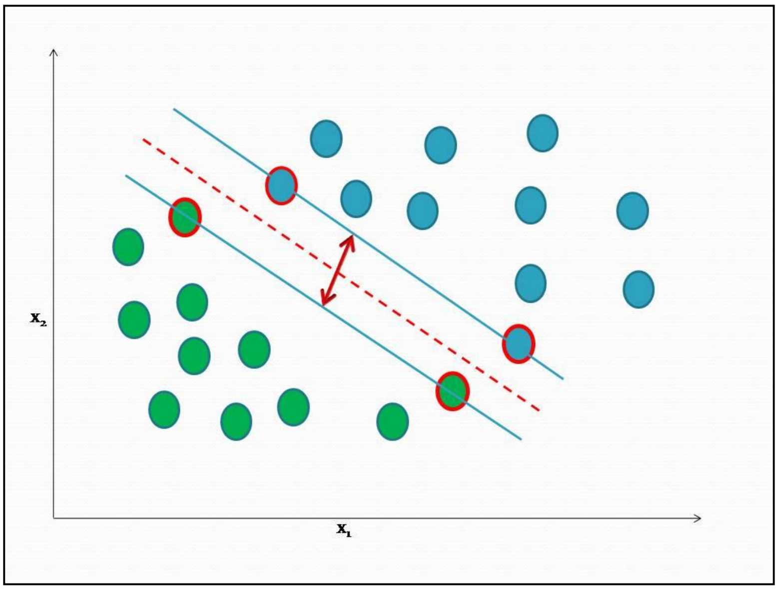

Figure 2. Optimal Hyperplane of Support Vector Machine.

In a machine learning framework, training may result in overfitting when the model produced is significantly affected by short-run dynamics and idiosyncratic features of the specific data sample at hand such as possible noise, instead of the true underlying long-run relationship that describes the phenomenon and not the sample. Usually, when overfitting occurs we observe a high performance (fit) on the training step and a significant accuracy drop on the out-of-sample testing step. Overfitting can be avoided by using a *k*-fold cross validation. The dataset is divided into *k* subsets and the training-testing steps are repeated *k* times. In each turn a different subset is used as the test dataset, while the rest *k*-1 ones are grouped together to form the training dataset. The procedure is called *dataset folding*. The selected model is the one that produces the highest average performance over all *k*-folds. In Figure 3 we present a graphical representation of a 3-fold cross validation procedure. The model's generalization ability is then tested using an out-of-sample set (a subset that did not participate in the cross-validation procedure).

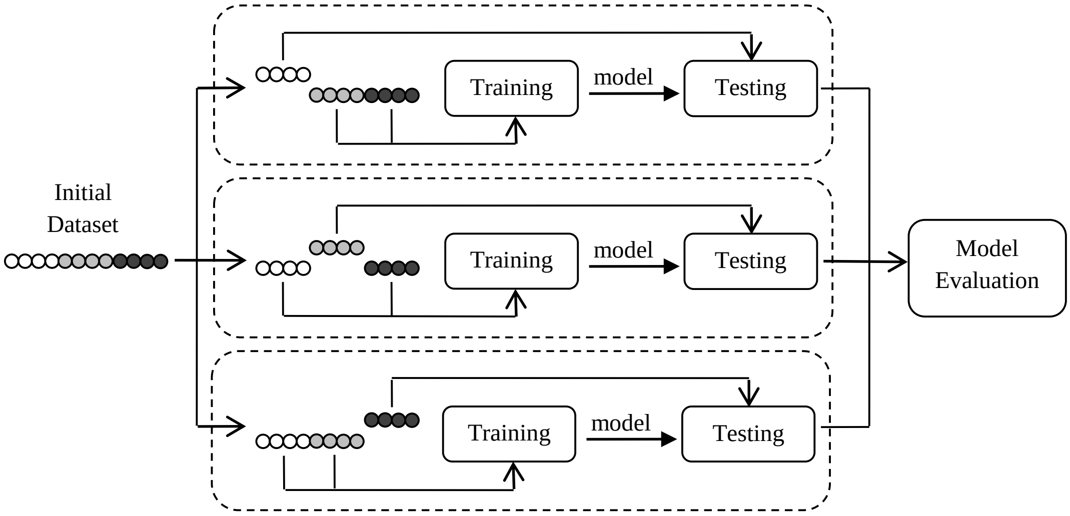

Figure 3. Overview of a 3-fold Cross Validation Evaluation System

For our purposes we used a 5-fold cross validation procedure. The described scheme was performed for four kernels: the linear, the radial basis function (RBF), the polynomial and the sigmoid. The mathematical representation of each kernel is:

Linear $$K_1(\boldsymbol{x}_1, \boldsymbol{x}_2) = \boldsymbol{x}_1^T \boldsymbol{x}_2 + r \tag{1}$$

RBF $$K_2(\boldsymbol{x}_1, \boldsymbol{x}_2) = e^{-\gamma \|x_1 - x_2\|^2} \tag{2}$$

Polynomial $$K_3(\boldsymbol{x}_1, \boldsymbol{x}_2) = (\gamma \boldsymbol{x}_1^T \boldsymbol{x}_2 + r)^d \tag{3}$$

Sigmoid $$K_4(\boldsymbol{x}_1, \boldsymbol{x}_2) = \tanh(\gamma \boldsymbol{x}_1^T \boldsymbol{x}_2 + r) \tag{4}$$

With factors d, r, γ representing kernel parameters that need to be optimized.

The search for the optimal parameter setup in every kernel case was performed in a coarse-to-fine grid search evaluation scheme. In this type of grid search, the parameters are initially evaluated in a large step grid in order to achieve a low accuracy image of the parameters’ performance. Then, we seek improved results using a denser grid focusing only in the parts of our search area where the model achieved a high performance. The procedure can be repeated multiple times. In Figure 4, we provide a graphical representation of a three-iteration coarse-to-fine grid search. Optimum results in terms of forecasting performance are depicted with gray color. As the area becomes darker, the grid step becomes smaller and the search finer. Coarse-to-fine grid search is a lower complexity bypass of the exhaustive search in the finer level.

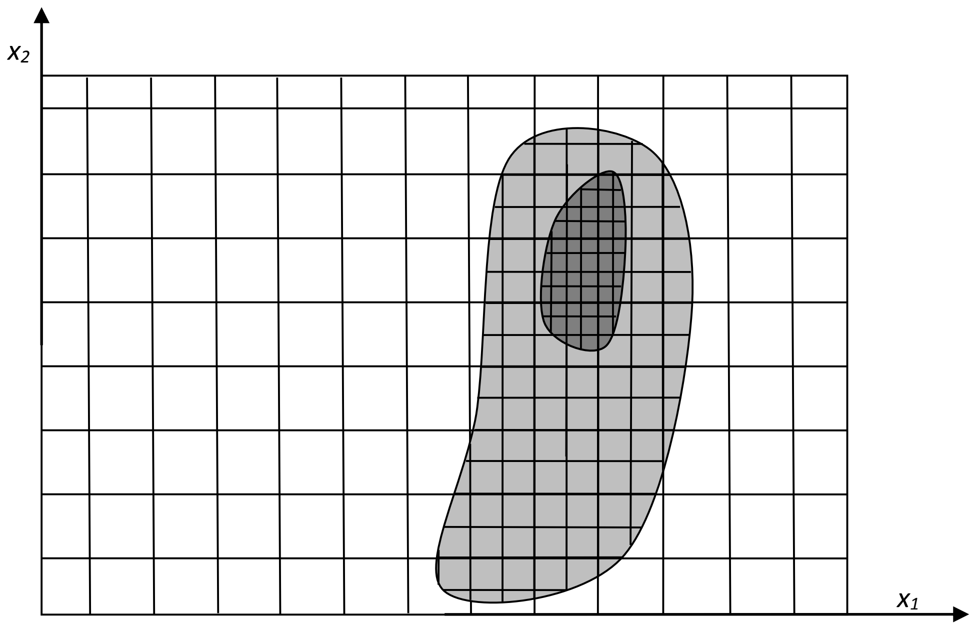

Figure 4. The coarse-to-fine grid search. From the coarser search, we advance to denser ones as the regions become darker (when darker represents better results).

2.2 The Probit Model

In this section we describe in brief the probit model, the well-established and widely employed econometric approach for binary classification.

The probit model is a regression that estimates the probability:

$$P_r(y=1|\boldsymbol{x},\boldsymbol{\beta}) = \Phi(\boldsymbol{\beta}^T\boldsymbol{x}) \qquad (5)$$

Where $P_r(y=1)$ is the probability that an event y will occur or not and is assumed to be determined by a set of independent variables provided in vector $\boldsymbol{x}$. y is a dummy dependent variable, taking the value of 1 or 0. In our case, a value of zero (0) indicates a GDP below trend (unemployment gap) and the value of one (1) indicates an economy above trend (inflationary gap). Φ denotes the Cumulative Distribution Function (CDF) of the standard normal distribution and $\boldsymbol{\beta}$ is the vector to be estimated.

3. Data and empirical results

The dataset used in this paper is composed of the seven most commonly watched U.S. federal Government Bonds and Treasury Bills interest rates and the real seasonally adjusted quarterly GDP of the U.S. spanning the period from July 1976 to October 2011.

Specifically, we use the Treasury Bills with maturities of 3 and 6 months and the U.S Government Bonds with maturities of 2, 3, 5, 7 and 10 years gathered from the database of the Federal Reserve Bank of Saint Louis (FRED), expressed at a quarterly frequency. We assume that the short term interest rates are the 3 months, 6 months and 2 years interest rates, while the long term interest rates are the 3, 5, 7 and 10 years interest rates. The data set is separated as follows: the data from 1976Q3 to 2006Q4 (122 observations) are fed in the cross-validation part of our scheme and the data from 2007Q1 to 2011Q4 (20 observations) are used for the out-of-sample evaluation of our models. The out-of-sample observations include 8 instances of unemployment gaps (recessions) and 12 cases of inflationary gaps (growth).

The real GDP figures were transformed into natural logarithms. For our purposes, recessions are defined as the output gaps, i.e. a real GDP value below its long-run trend. We employed the Hodrick-Prescott filter by setting the λ parameter equal to 1600, in order to decompose the real GDP variable into the trend and the cyclical component (Hodrick, Prescott 1997). The cyclical component series is then transformed into a binary (dummy) variable that has the value of +1 whenever the cyclical component is greater than zero (i.e. a real GDP value above the trend) and -1 elsewhere.

Overall, we tested a) all twelve combinations of short-term and long-term interest rate pairs and in order to enhance our models we include some information on the curvature of the yield as well using b) all thirty combinations of three interest rates (one short-term and two long-term interest rates or two short-term and one long-term interest rates) and c) all the interest rates together (both short-term and long-term).

3.1 Two interest rates input set

First we exhaustively tested all possible pairs of two interest rates: one long-term interest rate and one short-term interest rate. The results are presented in table 1.

Table 1. Forecasting Accuracy of SVM models (all kernels) and probit models on combinations of two interest rates.

	Interest Rates	Accuracy of in-sample Data	Accuracy of out-of-sample Data	Growth accuracy	Recession accuracy
Linear	2Y-7Y	63.33%	70%	50%	100%
RBF	2Y-3Y	77.50%	75%	58%	100%

Polynomial	3M-10Y	77.50%	80%	83%	75%
Sigmoid	2Y-10Y	70%	60%	100%	0%
Probit	3M-5Y	69%	65%	42%	100%
Probit	6M-5Y	69%	65%	42%	100%

The best overall accuracy on out-of-sample data was achieved when we combined the three months interest rate with the ten years interest rate using the polynomial kernel (80%). Nonetheless, taking into account that the main focus of our study was to forecast recessions (output gaps) we observe that the best such performance is achieved with the RBF kernel when fed with the two years interest rate and the three years interest rate. The model forecasted correctly 8 out of 8 recession cases (100% success) and 7 out of 12 inflationary gaps (58% success); the overall accuracy was thus 75%. We also achieved 100% recession forecasting accuracy with the linear kernel but with a lower out of sample accuracy (70%).

We adopted a similar scheme for the probit based models: We estimated the probit models using the first 122 observations covering the period from 1976Q3 to 2006Q4 and then we tested their forecasting ability in the out-of-sample data spanning from 2007:Q1 to 2011:Q4. The best forecasting accuracy was achieved with two input variable combinations: a) the 3 months-5 years and b) the 6 months-5 years pairs both yielding 100% recession forecasting, and an overall 65% forecasting accuracy. Thus, although both the SVM and probit models achieved 100% accuracy in recession forecasting, the SVM model was more accurate in forecasting inflationary gaps (58% accuracy) than the probit model (42% accuracy) and as a result overall as well (75% and 65% respectively).

3.2 Three interest rates input set

The standard approach in the relevant literature of forecasting economic activity through the yield curve is the use of only a pair of interest rates. Thus only the slope of the yield curve is taken into account. In our empirical work we also tried to explore the forecasting ability of the yield curve by including more information from it. First, we test all possible combinations of three interest rates in an effort to use not only the slope of the yield curve but its curvature as well. Thus, we tested models with all possible combinations of three interest rates, either in groups of one long-term interest rate with two short-term interest rates or in groups of two long-term interest rates with one short-term interest rate (Table 2).

Table 2. Forecasting accuracy of SVM models (all kernels) and probit models on combinations of three interest rates.

	Interest Rates	Accuracy of in-sample data	Accuracy of out-of-sample data	Growth accuracy	Recession Accuracy
Linear	6M-5Y-10Y	62.50%	70%	50%	100%
Linear	2Y-3Y-5Y	62.50%	70%	50%	100%
Linear	2Y-5Y-7Y	62.50%	70%	50%	100%
RBF	3M-2Y-3Y	76.67%	80%	67%	100%
Polynomial	3M-3Y-7Y	76.67%	75%	83%	63%
Polynomial	2Y-5Y-7Y	76.67%	75%	83%	63%
Polynomial	6M-7Y-10Y	76.67%	75%	83%	63%
Sigmoid	6M-5Y-10Y	70.83%	60%	100%	0%
Sigmoid	6M-7Y-10Y	70.83%	60%	100%	0%
Probit	2Y-5Y-7Y	71%	75%	58%	100%

According to the results presented in Table 2, the best overall out-of-sample accuracy using the combinations of three interest rates was achieved by the RBF kernel at 80%. Three models achieve 100% accuracy in recession forecasting: the SVM with the linear and the RBF kernels and the probit model; their accuracies in forecasting inflationary gaps are 50%, 67% and 58% respectively. The best overall out-of-sample accuracy, 80%, was achieved when we fed the three months, two years and three years interest rate on the SVM model coupled with the RBF kernel: 8 out of 8 output gaps (100% accuracy) and 8 out of 12 inflationary gaps (67% accuracy).

The probit model was competitive, but slightly outperformed in performance by the Machine Learning methodology, yielding 100% accuracy in unemployment gap forecasting, though just 58% forecasting accuracy in inflationary gaps resulting in an overall accuracy of 75%.

3.3 All interest rates input set

Based on Piazzesi and Wei (2004), next we included in our forecasting models all interest rates in our sample. Within this context we include all information provided by the yield curve, short-, mid- and long-term interest rates by effectively tracing the curvature of the yield curve. The empirical results are presented in Table 3.

Table 3. Forecasting accuracy of SVM models (all kernels) and probit models of all interest rates.

	Interest Rates	Accuracy of In-sample data	Accuracy of out-of-sample data	Growth accuracy	Recession accuracy
Linear	All	63%	65%	42%	100%
RBF	All	73%	70%	50%	100%
Polynomial	All	71%	70%	75%	63%
Sigmoid	All	67%	40%	67%	0%
Probit	All	71%	55%	25%	100%

The best out-of-sample accuracy was achieved in both the RBF and the polynomial kernels with 70% but while the RBF provides 100% accuracy in recession forecasting, the polynomial reaches only 63%. The linear kernel produces 100% accuracy in recession forecasting as well but its inflationary gap forecasting accuracy is only 42% when compared to the RBF's 50%. Thus, the RBF kernel forecasted correctly 8 out of 8 output gaps and 6 out of 12 cases of future inflationary gaps (50%). The results obtained from the probit models were inferior to those of the SVM models yielding just 55% overall accuracy.

Summarizing the above results, we conclude that: a) The SVM model with the RBF kernel and three interest rates as predictors achieved the best overall forecasting performance with 80% accuracy, yielding 100% out-of-sample forecasting accuracy of recessions (unemployment gaps) and 67% percent accuracy on inflationary gaps. b) The curvature of the yield curve appears to be important in forecasting economic activity: the model with three interest rates that takes into account the curvature achieved the best out-of-sample forecasting ability in contrast to the simple slope model that is widely used in the relevant literature. Nonetheless, the inclusion of all interest rates in the forecasting model reduces the overall forecasting ability of the model. c) Two interest rates, the two and three years bond rates appear in the best forecasting models using two and three interest rates. As our best overall model is the one that uses the curvature of the yield curve that is traced by the three-month T-bill rate and the two and three years bonds interest rates, we provide empirical evidence that only the short and medium term rates are adequate in forecasting recessions. Long term rates in most cases significantly reduce the forecasting ability of the models to signal a recession. Only with the SVM and the linear kernel the ten year rate provides a 100% recession forecasting but with a significant drop in inflationary gap forecasting to a mere 50%. A possible explanation for this result may be that in a developed industrialized country such as the U.S. the long-term outlook of the economy may be considered relatively stable. This long-run trend is not affected by the short-term dynamics and fluctuations so that agents' views on future economic activity remain tied to that trend and are not affected by short-term events or fluctuations from that trend.

4. Conclusion

In this paper we use two alternative models to examine the out-of-sample forecasting ability of the yield curve in terms of real output. We mainly focus in forecasting instances of a recession (an output gap). The innovation of our approach with respect to the vast relevant literature is that to the best of our knowledge this is the first time that the empirical framework of Support Vector Machines for classification is used in this problem. Moreover, we depart from standard analysis by not only using a pair of interest rates (the slope of the yield curve) but we also use the curvature of the yield curve to provide forecasts (three and all interest rates). We use four alternative kernels for the SVM estimations and compare our results to the standard and widely used in the relevant literature probit model. We use in our data sample seven alternative maturities of U.S. treasury bills and bonds and the real seasonally adjusted GDP.

Our empirical results show that the SVM model with the RBF kernel and three interest rates as input variables has the best overall forecasting ability with 80% accuracy in out-of-sample forecasting. This model achieves a 100% out-of-sample forecasting accuracy of recessions (unemployment gaps) and a 67% percent accuracy on inflationary gaps. Thus, it appears that the correct identification of upcoming unemployment gaps may be achieved at the cost of some inflation. The rates that produce these results are the three month T-bill rate and the two and three year government bond interest rates. Thus, the model that takes into account the curvature and not only the slope of the yield curve appears to have the best out-of-sample forecasting ability. Long term rates do not appear to complement the forecasting ability of our models. Our interpretation of this finding is that the U.S. being a developed industrialized country is considered to have a stable long-term economic outlook that is not affected by short-term dynamics and fluctuations but adheres to the long-run potential output. Thus, agents' views on future economic activity are not significantly affected by short-term events or fluctuations rendering the long-term rates uninformative.

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Appendix

Table 4. Forecasting Accuracy of linear kernel on all combinations of two interest rates.

Interest rates	Best c	Accuracy of In-sample data (%)	Accuracy of out-of-sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-3Y	0.88	56.67	40	0	100
3M-5Y	14.44	58.33	70	50	100
3M-7Y	9.63	59.16	70	50	100
3M-10Y	2.66	58.33	70	50	100
6M-3Y	0.19	60	40	50	100
6M-5Y	7.56	58.33	70	0	100
6M-7Y	31.39	58.33	70	50	100
6M-10Y	61	61.66	70	50	100
2Y-3Y	1.81	60.83	40	0	100
2Y-5Y	451	62.5	70	50	100
2Y-7Y	53.38	63.33	70	50	100
2Y-10Y	91	65	65	42	100

Table 5. Forecasting Accuracy of linear kernel on all combinations of three interest rates.

Interest rates	Best c	Accuracy of In-sample data (%)	Accuracy of out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-6M-3Y	0.12	63.33	40	0	100
3M-6M-5Y	6.09	58.33	70	50	100
3M-6M-7Y	311	58.33	60	50	75
3M-6M-10Y	501	60	70	50	100
3M-2Y-3Y	0.7	57.5	40	0	100
3M-2Y-5Y	51	61.67	65	40	100
3M-2Y-7Y	151	63.33	65	42	100
3M-2Y-10Y	15.5	62.5	65	42	100
3M-3Y-5Y	221	64.17	65	42	100
3M-3Y-7Y	121	63.33	65	42	100
3M-3Y-10Y	71	64.17	65	42	100
3M-5Y-7Y	11	58.33	70	50	100
3M-5Y-10Y	161	60.83	65	42	100
3M-7Y-10Y	711	60.83	65	42	100
6M-2Y-3Y	0.75	56.67	40	0	100
6M-2Y-5Y	111	62.5	65	42	100
6M-2Y-7Y	51	64.16	65	42	100
6M-2Y-10Y	51	62.5	65	42	100
6M-3Y-5Y	251	63.33	65	42	100
6M-3Y-7Y	461	65	65	42	100
6M-3Y-10Y	28.19	62.5	65	42	100

6M-5Y-7Y	161	59.17	70	50	100
6M-5Y-10Y	51	62.5	70	50	100
6M-7Y-10Y	561	60.83	65	42	100
2Y-3Y-5Y	51	62.5	70	50	100
2Y-3Y-7Y	411	65	65	42	100
2Y-3Y-10Y	601	65	65	42	100
2Y-5Y-7Y	18.38	62.5	70	50	100
2Y-5Y-10Y	51	63.33	65	42	100
2Y-7Y-10Y	53.37	63.33	65	42	100

Table 6. Forecasting Accuracy of RBF kernel on all combinations of two interest rates.

Interest rates	Best c	Best g	Accuracy of In-sample data (%)	Accuracy of out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-3Y	621	1	68.33	70	50	100
3M-5Y	1091	1	72.5	70	50	100
3M-7Y	941	1	74.17	70	58.33	87.50
3M-10Y	223.83	1.45	75.83	70	50	100
6M-3Y	3.81	0.63	75	70	50	100
6M-5Y	320.5	1.25	73.33	70	50	100
6M-7Y	597.95	1.06	74.16	70	50	100
6M-10Y	1311	1	74.16	70	50	100
2Y-3Y	936.69	1.062	77.5	75	58.33	100
2Y-5Y	1031	1	78.33	70	58.33	87.50
2Y-7Y	1611	1	73.33	65	66.67	62.50
2Y-10Y	645.86	1.2	76.67	65	58.33	75

Table 7. Forecasting Accuracy of RBF kernel on all combinations of three interest rates.

Interest rates	Best c	Best g	Accuracy of In-sample data (%)	Accuracy of out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-6M-3Y	708.68	0.6875	73.33	65	50	87.50
3M-6M-5Y	30.81	1.3613	74.17	70	50	100
3M-6M-7Y	618.06	0.6875	75.83	70	58.33	87.50
3M-6M-10Y	101	1	78.33	65	50	87.50
3M-2Y-3Y	1.18	0.875	76.67	80	66.67	100
3M-2Y-5Y	441	1	78.33	70	58.33	87.50
3M-2Y-7Y	93.66	1.41	75	70	66.67	75
3M-2Y-10Y	36.39	1.6892	78.33	70	50	100
3M-3Y-5Y	241	1	77.5	65	50	87.50
3M-3Y-7Y	465.65	0.768	75	65	58.33	75
3M-3Y-10Y	161	1	75	65	50	87.50
3M-5Y-7Y	140.875	1.0625	71.67	70	50	100

3M-5Y-10Y	60.74	1.3821	75.83	60.74	50	100
3M-7Y-10Y	44.75	1.5231	75.83	70	50	100
6M-2Y-3Y	367.875	1.4375	75	70	50	100
6M-2Y-5Y	291	1	80	65	50	87.50
6M-2Y-7Y	350.64	0.8425	75.83	70	66.67	75.00
6M-2Y-10Y	47.75	1.4375	76.67	65	50	87.50
6M-3Y-5Y	984.47	0.7417	78.33	65	50	87.50
6M-3Y-7Y	332.0625	0.875	73.33	65	58.33	75
6M-3Y-10Y	331.46	1.25	75	65	66.67	62.50
6M-5Y-7Y	72.7	1.4063	73.33	70	50	100
6M-5Y-10Y	90.66	1.25	75	70	50	100
6M-7Y-10Y	26.1064	1.5485	75.83	65	58.33	75
2Y-3Y-5Y	186.31	1.0625	76.67	65	50	87.50
2Y-3Y-7Y	94.125	1.25	75	70	58.33	87.50
2Y-3Y-10Y	1	0.7263	76.67	65	58.33	75
2Y-5Y-7Y	350.49	0.875	77.5	70	58.33	87.50
2Y-5Y-10Y	251	1	75.83	65	50	87.50
2Y-7Y-10Y	36.14	1.6851	76.67	65	50	87.50

Table 8. Forecasting Accuracy of Sigmoid kernel on all combinations of two interest rates.

Interest rates	Best c	Best g	Best coef.	Accuracy of In-sample data (%)	Accuracy of out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-3Y	801	376	-270	69.16	35	58.33	0
3M-5Y	1	639	-438.75	70	35	58.33	0
3M-7Y	0.5332	23.9685	-13.33	69.16	40	66.67	0
3M-10Y	8.97	275.79	-163.23	69.17	40	66.67	0
6M-3Y	4.0625	34.275	-20.98	69.16	35	58.33	0
6M-5Y	0.5359	1.223	-1.02	68.33	25	16.67	37.50
6M-7Y	1	1001	180	69.17	35	50	12.50
6M-10Y	1.03	300.5	-168.75	67.5	35	50	12.50
2Y-3Y	0.5536	192.1228	86.0251	67.5	60	100	0
2Y-5Y	37.75	310.0625	144.375	67.5	40	0	100
2Y-7Y	451	576	-240	67.5	60	100	0
2Y-10Y	6.3125	203.8477	-86.1328	70	60	100	0

Table 9. Forecasting Accuracy of Sigmoid kernel on all combinations of three interest rates.

Interest rates	Best c	Best g	Best coef.	Accuracy of In-sample data (%)	Accuracy of out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-6M-3Y	1	151	-150	70.83	35	58	0
3M-6M-5Y	0.5156	120.75	-118.125	70.83	35	58	0

3M-6M-7Y	23.5	69.0594	-70.6289	70.83	35	58	0
3M-6M-10Y	0.5156	129.3359	-128.906	70.83	35	58	0
3M-2Y-3Y	47.8812	6.745	-5.6396	70	35	58	0
3M-2Y-5Y	1.625	96.7723	-92.5313	68.33	35	58	0
3M-2Y-7Y	1	176	-180	67.5	35	58	0
3M-2Y-10Y	451	926	240	70	60	100	0
3M-3Y-5Y	79.3	269.3971	71.6799	68.33	60	100	0
3M-3Y-7Y	0.5298	102.5117	-97.5	66.66	40	67	0
3M-3Y-10Y	6.5	95	-75	67.5	40	67	0
3M-5Y-7Y	1.1863	75.1453	-49.0078	70	45	75	0
3M-5Y-10Y	0.2578	175.2803	-123.267	69.1667	55	92	0
3M-7Y-10Y	101	251	-180	70	60	100	0
6M-2Y-3Y	1	726	180	67.5	35	58	0
6M-2Y-5Y	0.5	1.7195	-1.647	66.67	35	58	0
6M-2Y-7Y	1	676	180	66.67	60	100	0
6M-2Y-10Y	1	501	-300	67.5	40	67	0
6M-3Y-5Y	176	101	0	68.33	60	100	0
6M-3Y-7Y	25.0625	30.7663	-20.6543	70	40	67	0
6M-3Y-10Y	4.75	171.8063	-108.281	67.5	45	75	0
6M-5Y-7Y	1.625	54.5813	-34.5938	71.67	50	83	0
6M-5Y-10Y	1.8688	67.3065	-45.3186	70.83	60	100	0
6M-7Y-10Y	15.9375	86.625	-61.875	70.83	60	100	0
2Y-3Y-5Y	1.625	362.977	18.1187	67.5	40	0	100
2Y-3Y-7Y	0.5862	1.5337	-628.418	66.67	40	67	0
2Y-3Y-10Y	137.75	137.75	108.75	71.67	40	0	100
2Y-5Y-7Y	10.1	162.32	124.8618	68.33	40	0	100
2Y-5Y-10Y	3.1875	156.532	-99.9141	68.33	60	100	0
2Y-7Y-10Y	1.625	66.004	-43.808	69.1667	60	100	0

Table 10. Forecasting Accuracy of polynomial kernel on all combinations of two interest rates.

Interest rates	**Best d**	**Best c**	**Best g**	**Best coef.**	**Accuracy of In-sample data (%)**	**Accuracy of out-of sample data (%)**	**Growth Accuracy (%)**	**Recession Accuracy (%)**
3M-3Y	6	4.81	2.23	0.5197	70.83	70	50	100
3M-5Y	3	0.8125	4.0921	0.5188	71.67	75	75	100
3M-7Y	3	40.375	1	1.625	75	80	83.33	100
3M-10Y	3	352.34	2	0.25	77.5	80	83.33	100
6M-3Y	6	6.1096	2.3079	0.4533	75	70	0.5	100
6M-5Y	3	8.125	1.4375	0.4297	72.5	80	83.33	100
6M-7Y	3	0.8125	5.5756	0.7375	74.16	70	75	100
6M-10Y	3	24.375	1.375	1.0625	75.83	75	83.33	100
2Y-3Y	7	10.71	0.938	1.4517	79.16	70	58.33	100
2Y-5Y	6	86.625	1.0625	0.6563	78.33	65	50	100
2Y-7Y	3	25.125	1.46	0.2422	76.67	75	83.33	100
2Y-10Y	7	102	0.6875	1.625	77.5	65	50	100

Table 11. Forecasting Accuracy of polynomial kernel on all combinations of three interest rates.

Interest rates	Best d	Best c	Best g	Best coef.	Accuracy of In-sample data (%)	Accuracy out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-6M-3Y	6	106.50	1.06	2.04	76.67	60	50	75
3M-6M-5Y	6	2.22	1.59	2.38	77.50	60	50	75
3M-6M-7Y	6	5.78	1.31	1.41	77.50	60	50	75
3M-6M-10Y	3	11.98	1.78	3.62	78.33	60	66.67	50
3M-2Y-3Y	6	6.35	0.95	1.75	80.83	70	58.33	87.50
3M-2Y-5Y	7	4.22	1.36	1.06	77.50	65	50	87.50
3M-2Y-7Y	6	5.31	1.05	1.38	75.00	70	58.33	87.50
3M-2Y-10Y	8	1.63	0.69	1.44	75.83	65	50	87.50
3M-3Y-5Y	8	17.27	0.44	1.44	78.33	65	50	87.50
3M-3Y-7Y	3	3.25	5.66	0.43	76.67	75	83.33	62.50
3M-3Y-10Y	8	0.25	1.06	1.83	75.00	75	66.67	87.50
3M-5Y-7Y	3	1.94	5.41	0.62	75.83	65	66.67	62.50
3M-5Y-10Y	3	0.81	4.92	1.20	75.00	60	50	75
3M-7Y-10Y	7	1.20	0.93	1.06	74.16	60	50	75
6M-2Y-3Y	6	25.53	0.66	1.89	80.83	65	58.33	75
6M-2Y-5Y	7	1.21	1.26	1.70	77.50	70	58.33	87.50
6M-2Y-7Y	6	32.50	1.06	0.88	74.16	70	58.33	87.50
6M-2Y-10Y	7	0.25	1.41	2.04	75.83	70	58.33	87.50
6M-3Y-5Y	7	33.33	1.01	1.92	77.50	70	50	100
6M-3Y-7Y	6	3.27	1.68	1.71	73.33	70	58.33	87.50
6M-3Y-10Y	6	364.31	0.88	1.06	75.00	70	50	100
6M-5Y-7Y	7	0.10	2.92	0.82	75.83	70	66.67	75
6M-5Y-10Y	3	2.95	1.28	4.66	76.67	70	66.67	75
6M-7Y-10Y	3	6.50	1.20	1.25	76.67	75	83.33	62.50
2Y-3Y-5Y	7	0.25	1.06	2.16	77.50	65	50	87.50
2Y-3Y-7Y	3	4.72	3.44	2.54	75.00	70	83.33	50
2Y-3Y-10Y	3	0.81	1.83	0.73	75.83	65	66.67	62.50
2Y-5Y-7Y	3	2.19	2.00	0.71	76.67	75	83.33	62.50
2Y-5Y-10Y	3	12.36	13.75	0.92	75.83	65	75	50
2Y-7Y-10Y	3	0.81	3.27	0.45	75.83	65	66.67	62.50

Table 12. Forecasting Accuracy of probit models on all combinations of two interest rates.

Interest rates	Accuracy of In-sample data (%)	Accuracy out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-3Y	63	65	42	100
3M-5Y	69	65	42	100
3M-7Y	65	65	42	100
3M-10Y	67	65	42	100
6M-3Y	64	65	42	100

6M-5Y	69	65	42	100
6M-7Y	67	65	42	100
6M-10Y	67	65	42	100
2Y-3Y	68	65	42	100
2Y-5Y	68	60	33	100
2Y-7Y	69	60	33	100
2Y-10Y	69	60	33	100

Table 13. Forecasting Accuracy of probit models on all combinations of three interest rates.

Interest rates	Accuracy of In-sample data (%)	Accuracy out-of sample data (%)	Growth Accuracy (%)	Recession Accuracy (%)
3M-6M-5Y	69,92	70	87.50	58.33
3M-6M-7Y	67.48	70	87.50	58.33
3M-6M-10Y	67.48	65	100	41.67
3M-2Y-3Y	68.29	55	100	25
3M-2Y-5Y	69.92	60	100	33.33
3M-2Y-7Y	69.11	60	100	33.33
3M-2Y-10Y	68.29	60	100	33.33
3M-3Y-5Y	66.67	60	100	33.33
3M-3Y-7Y	68.29	60	100	33.33
3M-3Y-10Y	68.29	60	100	33.33
3M-5Y-7Y	65.04	65	100	41.67
3M-5Y-10Y	68.29	60	100	33.33
3M-7Y-10Y	66.67	55	100	25
6M-2Y-3Y	66.67	65	100	41.67
6M-2Y-5Y	69.11	60	100	33.33
6M-2Y-7Y	69.11	60	100	33.33
6M-2Y-10Y	68.29	60	100	33.33
6M-3Y-5Y	67.48	60	100	33.33
6M-3Y-7Y	68.29	60	100	33.33
6M-3Y-10Y	68.29	60	100	33.33
6M-5Y-7Y	69.11	65	100	41.67
6M-5Y-10Y	68.29	60	100	33.33
6M-7Y-10Y	66.67	55	100	25
2Y-3Y-5Y	67.48	60	100	33.33
2Y-3Y-7Y	68.29	60	100	33.33
2Y-3Y-10Y	68.29	60	100	33.33
2Y-5Y-7Y	70.73	75	100	58.33
2Y-5Y-10Y	68.29	60	100	33.33
2Y-7Y-10Y	66.67	55	100	25